9 – Probability Part (1)

Combinatorics 1M020

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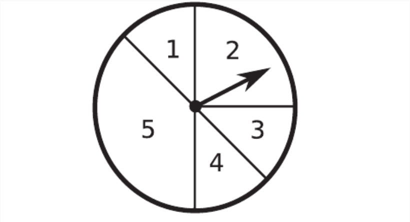
Introduction

A game of change – spinner

If we give the arrow a push



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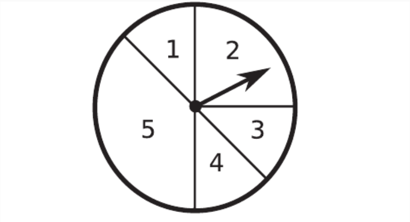

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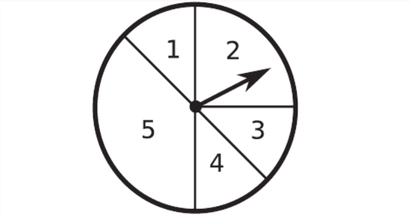

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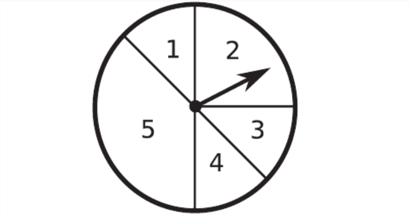

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- When we land in an odd numbered region, then 60% of the time, it will be in region 5.

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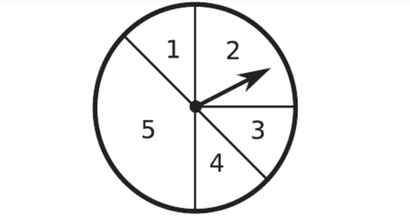

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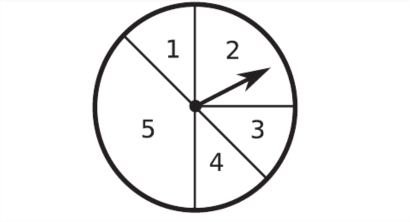
For $E \subseteq S$, $P(E)$ is the probability of E.

Example – spinner

Spinner

For the spinner, we can take $S = \{1, 2, 3, 4, 5\}$, with $P(1) = P(3) = P(4) = 1/8, P(2) = 2/8 = 1/4$ and $P(5) = 3/8$.
So $P(\{2, 3\}) = 1/8 + 2/8 = 3/8$.



Example – die

A six sided die

We can take $S = \{1, 2, 3, 4, 5, 6\}$ with $P(i) = 1/6$ for all $i \in S$.
So $P(\{2, 4\}) = ?$.

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Let's throw two dice.

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If we only care the sum of the dots on the top faces, we can take $S = \{2, 3, \dots, 12\}$ with

$$\begin{aligned} P(2) &= P(12) = 1/36, & P(3) &= P(11) = 2/36, \\ P(4) &= P(10) = 3/36, & P(5) &= P(9) = 4/36, \\ P(6) &= P(8) = 5/36, & P(7) &= 6/36 \end{aligned}$$

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How do we get $P(9)$?

- Among $6 \times 6 = 36$ combinations of the two dice,
- four $(3, 6), (4, 5), (5, 4), (6, 3)$ add up to 9.

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Answer

$$P(i) = \frac{\binom{9}{i}\binom{11}{3-i}}{\binom{20}{3}}$$

for $i = 0, 1, 2, 3$, while $P(4) = P(5) = 0$.

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The probability of a full house is

$$\frac{13 \times 12 \times \binom{4}{3} \times \binom{4}{2}}{\binom{52}{5}}$$

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The probability of A given B, denoted by $P(A|B)$ is

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So

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{9\times 6}{20\times 19}}{\frac{9}{20}} = \frac{6}{19} > \frac{6}{20} = P(B)$$

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$$\begin{aligned} P(G) &= P(G \cap J_1) + P(G \cap J_2) \\ &= P(G|J_1)P(J_1) + P(G|J_2)P(J_2) = \frac{1}{2}\frac{\binom{5}{2}}{\binom{20}{2}} + \frac{1}{2}\frac{\binom{4}{2}}{\binom{18}{2}} \end{aligned}$$

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A pair of dice

A pair of dice are rolled, one red and one blue. Let A be the event that the red die shows either a 3 or a 5, and let B be the event that both red die and the blue die show the same number. Then $P(A) = 2/6$, $P(B) = 6/36$, and $P(A \cap B) = 2/36$. So A and B are independent.

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So $P(A \cap B) \neq P(A)P(B)$, i.e., A and B are dependent.

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Consider a game in which you roll two dice, and you win if the total is 7 and you lose if the total is 2,3. You keep rolling until one of these totals occurs. Is this a fair game?

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Let B be the even that the sum is in $\{2, 3, 7\}$. Then

$$P(B) = P(2) + P(3) + P(7) = \frac{1}{36} + \frac{2}{36} + \frac{6}{36} = \frac{9}{36}$$

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$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{6/36}{9/36} = \frac{2}{3}.$$

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You are dealt with a hand of 5 card from a normal shuffled pack of 52 cards.

What is the probability that you get exactly two kings and one ace (event $A \cap B$)?

$$P(A \cap B) = \frac{\binom{4}{2}\binom{4}{1}\binom{44}{2}}{\binom{52}{4}}$$

What is the probability that you get exactly one ace (event $A \cap B$) given that you get exactly two kings?

$$P(A|B) = P(A \cap B)/P(B) = \frac{\frac{\binom{4}{2}\binom{4}{1}\binom{44}{2}}{\binom{52}{4}}}{\frac{\binom{4}{2}\binom{48}{3}}{\binom{52}{4}}} = \frac{\binom{4}{1}\binom{44}{2}}{\binom{48}{3}} = \frac{473}{2162}$$

Appendix

Self-study guide (for people who missed the class)

- Watch online video lectures 1-5 here.
- Read textbook chapter 10.1-10.2